Chapter 11 Series Part 1

1. The first 3 terms in the expansion of $(3 - ax)^5$, in ascending powers of x, can be written in the form $b - 81x + cx^2$. Find the value of each of a, b and c. [5]

$(3-ax)^5 = 3^5 - {}^5C_1 \times 3^4 \times ax + {}^5C_2 \times 3^3 \times a^2x^2$

$= 243 - 405ax + 270a^2x^2$

$b = 243$, $405a = 81$ $\quad 270a^2 = c$

$a = \frac{1}{5}$ $\quad \frac{270}{25} = c$

$c = \frac{54}{5}$

2. (a) Find the first 3 terms in the expansion of $(4 - \frac{x}{16})^6$ in ascending powers of *x*. Give each term in its simplest form.

[3]

$$\left(4-\frac{x}{16}\right)^6 = 4^6 - {}^6C_1 \times 4^5 \times \frac{x}{16} + {}^6C_2 \times 4^4 \times \frac{x^2}{256}$$

$$= 4096 - 384x + 15x^2$$

(b) Hence find the term independent of *x* in the expansion of $(4 - \frac{x}{16})^6 (x - \frac{1}{x})^2$.

[3]

$$\left(4096 - 384x + 15x^2\right)\left(x^2 - 2 + \frac{1}{x^2}\right)$$

$$= 4096 \times -2 + 15$$

$$= -8177$$

3. (a) Expand $(2-x)^5$, simplifying each coefficient.

$= 2^5 - {}^5C_1 \times 2^4 \times x + {}^5C_2 \times 2^3 \times x^2 - {}^5C_3 \times 2^2 \times x^3 + {}^5C_4 \times 2 \times x^4 - x^5$ [3]

$= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

(b) Hence solve $\dfrac{e^{(2-x)^5} \times e^{80x}}{e^{10x^4+32}} = e^{-x^5}$.

[4]

$$\frac{e^{32-80x+80x^2-40x^3+10x^4-x^5} \times e^{80x}}{e^{10x^4+32}} = e^{-x^5}$$

$$e^{\cancel{32}-\cancel{80x}+80x^2-40x^3+\cancel{10x^4}-x^5+\cancel{80x}-\cancel{10x^4}-\cancel{32}} = e^{-x^5}$$

$80x^2 - 40x^3 - x^5 = -x^5$

$40x^3 - 80x^2 = 0$

$x^3 - 2x^2 = 0$

$x^2(x-2) = 0$

$x^2 = 0$ or $x = 2$

$x = 0$

4. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

(a) Find the term independent of *x* in the binomial expansion of $(3x - \frac{1}{x})^6$. [2]

$(3x)^6 - {}^6C_1(3x)^5 \times (\frac{1}{x}) + {}^6C_2(3x)^4(\frac{1}{x})^2 - {}^6C_3(3x)^3(\frac{1}{x})^3$

term independent $= -540$

(b) In the expansion of $(1 + \frac{x}{2})^n$ the coefficient of x^4 is half the coefficient of x^6.
Find the value of the positive constant *n*. [6]

$(1 + \frac{x}{2})^n = 1^n + {}^nC_1 \times \frac{x}{2} + {}^nC_2 \frac{x^2}{2^2}$

${}^nC_4 \times \frac{1}{2^4} = \frac{1}{2}\, {}^nC_6 \times \frac{1}{2^6}$

$\frac{\cancel{n!}}{(n-4)! \times 4!} \times \frac{1}{\cancel{16}\ 1} = \frac{1}{2} \times \frac{\cancel{n!}}{(n-6)! \times 6!} \times \frac{1}{\cancel{64}\ 4}$

$\frac{1}{(n-4)(n-5)\cancel{(n-6) \times \ldots \times} \times 24} = \frac{1}{2} \times \frac{1}{\cancel{(n-6)}! \times 720 \times 4}$

$\frac{1}{(n-4)(n-5)} = \frac{24}{2 \times 720 \times 4}$

$\frac{1}{(n-4)(n-5)} = \frac{1}{240}$

$(n-4)(n-5) = 240$

$n^2 - 5n - 4n + 20 - 240 = 0$

$n^2 - 9n - 220 = 0$

$(n-20)(n+11) = 0$

$n = 20$ or $n = -11$ (reject)

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5. Find the coefficient of x^2 in the expansion of $(x - \frac{3}{x})(x + \frac{2}{x})^5$. [5]

$$x^5 + {}^5C_1 \times x^4 \times \frac{2}{x} + {}^5C_2 \times x^3 \times \frac{4}{x^2} + {}^5C_3 \times x^2 \times \frac{8}{x^3} + {}^5C_4 \times x \times \frac{16}{x^4} + \frac{32}{x^5}$$

$$= x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}$$

$$(x - \frac{3}{x})(x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5})$$

coe of $x^2 = 40 - 3 \times 10$

$= 10$

6. Given that the coefficient of x^2 in the expansion of $(1+x)(1-\frac{x}{2})^n$ is $\frac{25}{4}$, find the value of the positive integer n.

[5]

$(1+x)(1-\frac{x}{2})^n$

$1^n - {}^nC_1 \times \frac{x}{2} + {}^nC_2 \frac{x^2}{4} - \ldots$

$(1+x)(1 - {}^nC_1 \times \frac{x}{2} + {}^nC_2 \times \frac{x^2}{4})$

coe of $x^2 = {}^nC_2 \times \frac{1}{4} - {}^nC_1 \times \frac{1}{2}$

$\frac{25}{4} = \frac{n!}{(n-2)! \times 2! \times 4} - \frac{n!}{(n-1)! \times 2}$

$\frac{25}{4} = \frac{n!}{(n-2)! \times 8} - \frac{n!}{(n-1)! \times 2}$

$(\times 8)$

$50 = \frac{n!}{(n-2)!} - \frac{4n!}{(n-1)!}$

$50 = \frac{n \times (n-1) \cancel{(n-2) \times \ldots \times 1}}{\cancel{(n-2) \times \ldots \times 1}} - \frac{4(n \times \cancel{(n-1) \times \ldots \times 1})}{\cancel{(n-1) \times \ldots \times 1}}$

$50 = n^2 - n - 4n$

$0 = n^2 - 5n - 50$

$(n-10)(n+5) = 0$

$n = 10$ or $n = -5$ (Reject)

7. The first three terms in the expansion of $(a + bx)^5(1 + x)$ are $32 - 208x + cx^2$. Find the value of each of the integers a, b and c.

[7]

$(a^5 + {}^5C_1 \times a^4 \times bx + {}^5C_2 \times a^3 \times b^2x^2)(1+x)$

$= (a^5 + 5a^4bx + 10a^3b^2x^2)(1+x)$

$32 - 208x + cx^2 = a^5 + a^5x + 5a^4bx + 5a^4bx^2 + 10a^3b^2x^2$

$a^5 = 32$

$a = 2$

$a^5 + 5a^4b = -208$

$32 + 80b = -208$

$80b = -240$

$b = -3$

$5a^4b + 10a^3b^2 = c$

$-240 + 720 = c$

$c = 480$